CM2035 Topic 10 Graph Algorithms

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UNIVERSITY OF LONDON

Recap

In the module videos we have seen

- Ways of representing graphs.
- Minimum Spanning Tree (MST) algorithms (Prim's and Kruskal's).
- Shortest Path algorithms (Djikstra's)

A different algorithm

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An algorithm to determine whether there is a path between two nodes in the graph

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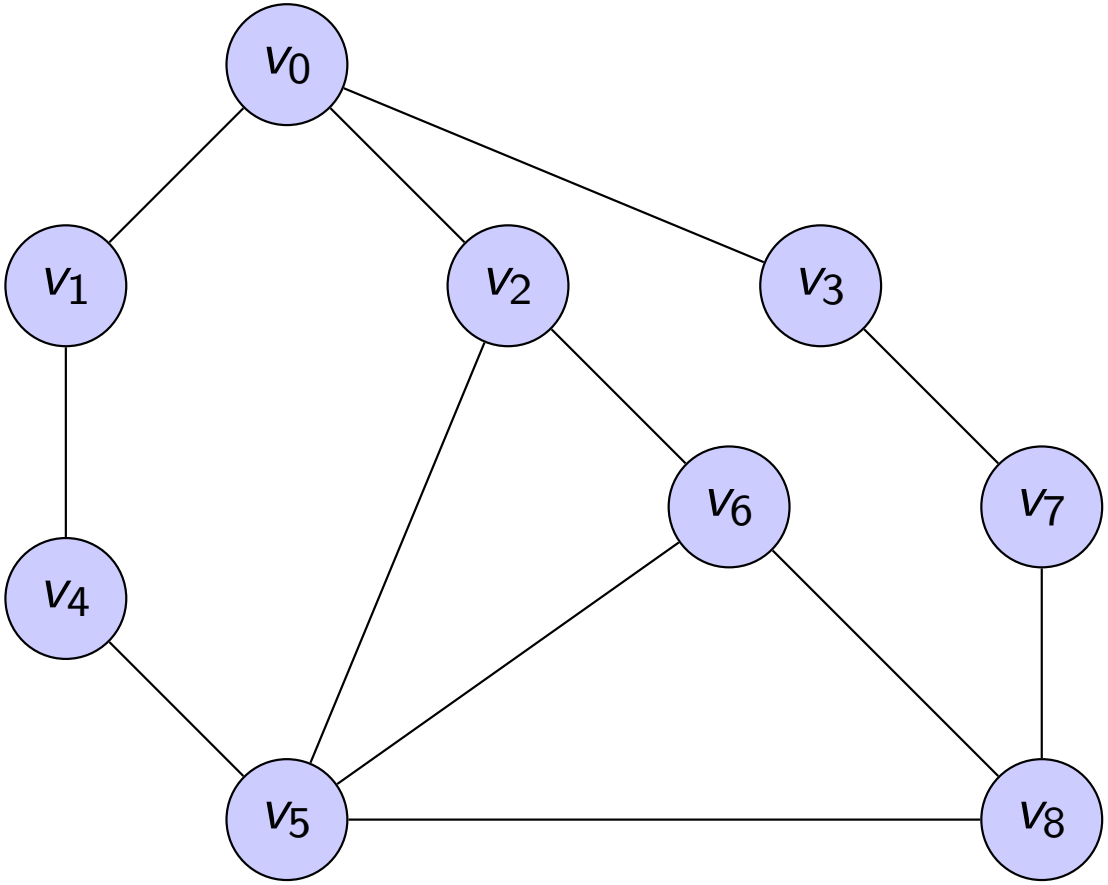
1. To reinforce the ideas behind dealing with graphs.
2. To relook at depth first search (and breadth first search) but on graphs rather than trees (as we considered before).

The question

Definition: A path is a sequence of non-repeated nodes $v_m, v_l, \ldots, v_k$ connected through edges present in a graph

Question: Given a graph $G(V, E)$ is there a path between v_i and v_j in G?

Example



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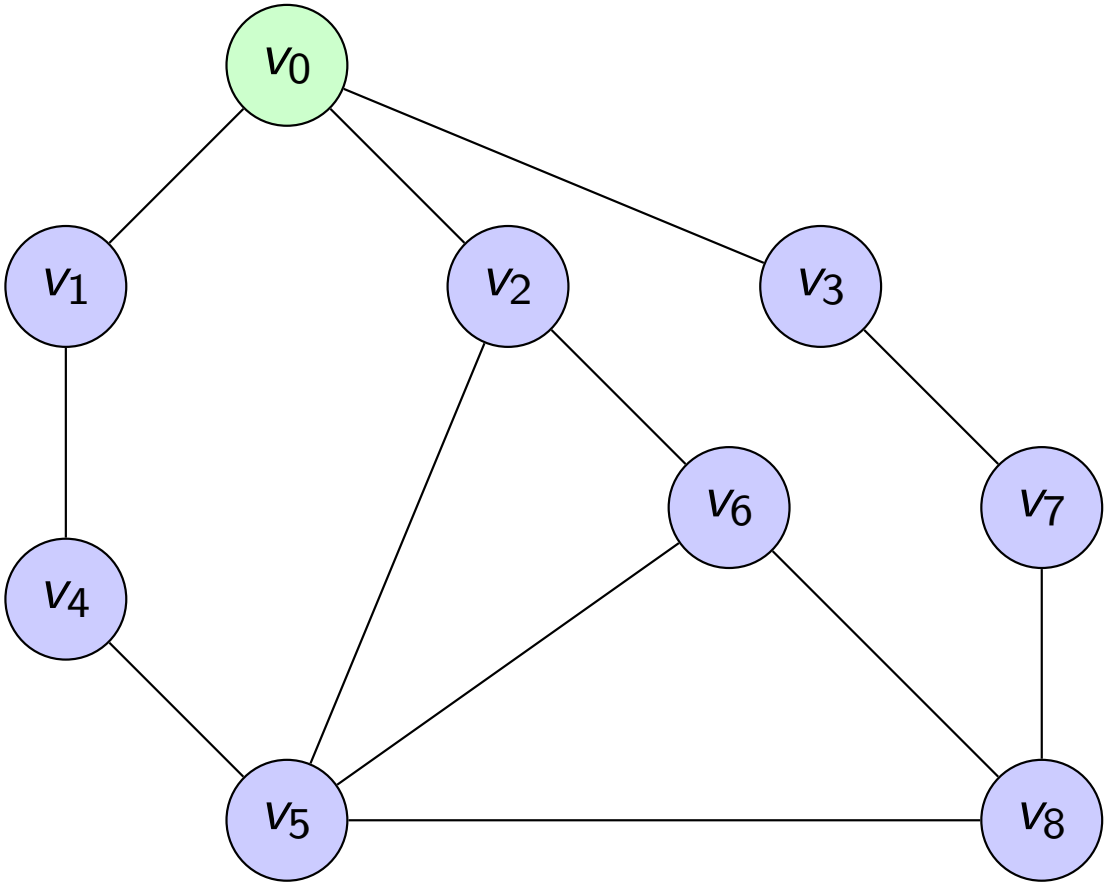
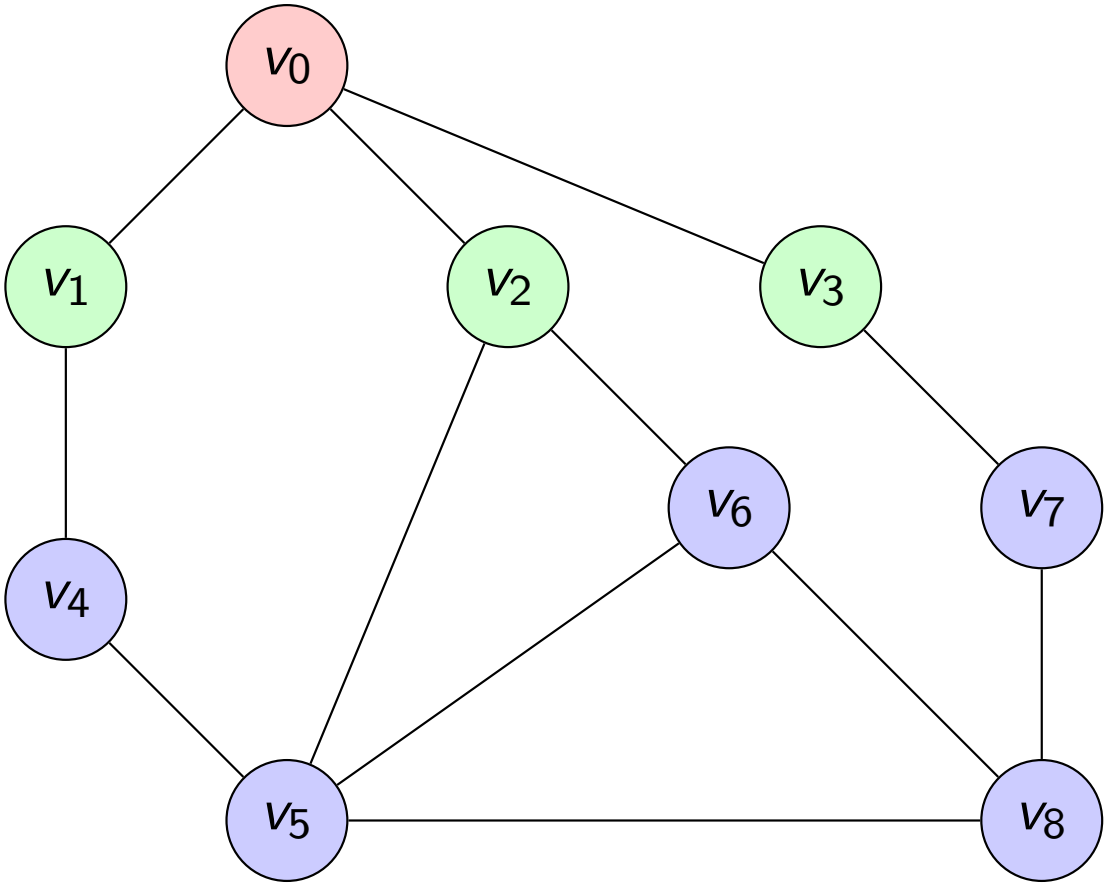
We start by making the `Frontier` our start node and setting `explored` to be empty.



Frontier $= \{v_0\}$ and explored $= \{\}$

The first node in the `Frontier` is not our goal so we add it to `explored` and then *expand* it.
Here *expanding* means finding its children and pushing them onto `Frontier` as we find them provided they have not already been explored and are not already in `Frontier`.



`Frontier` $= \{v_3, v_2, v_1\}$ and `explored` $= \{v_0\}$

We continue our search by

Checking if `Frontier` is empty. If it is then return *failure*.

Pop the first node from the stack

If it is v_4 then we have found a path return *success*

If it is not then add that node to the explored list and expand that node

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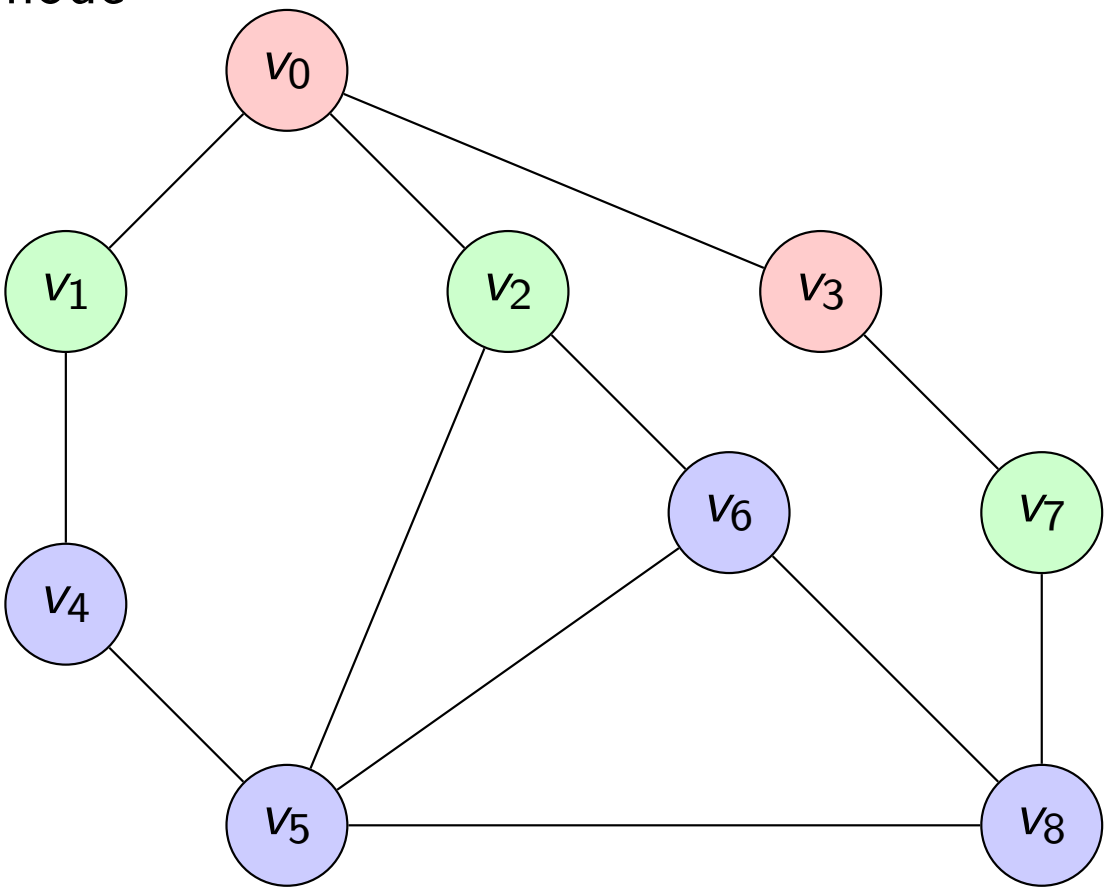

`Frontier` $= \{v_7, v_2, v_1\}$ and `explored` $= \{v_0, v_3\}$

We continue our search by

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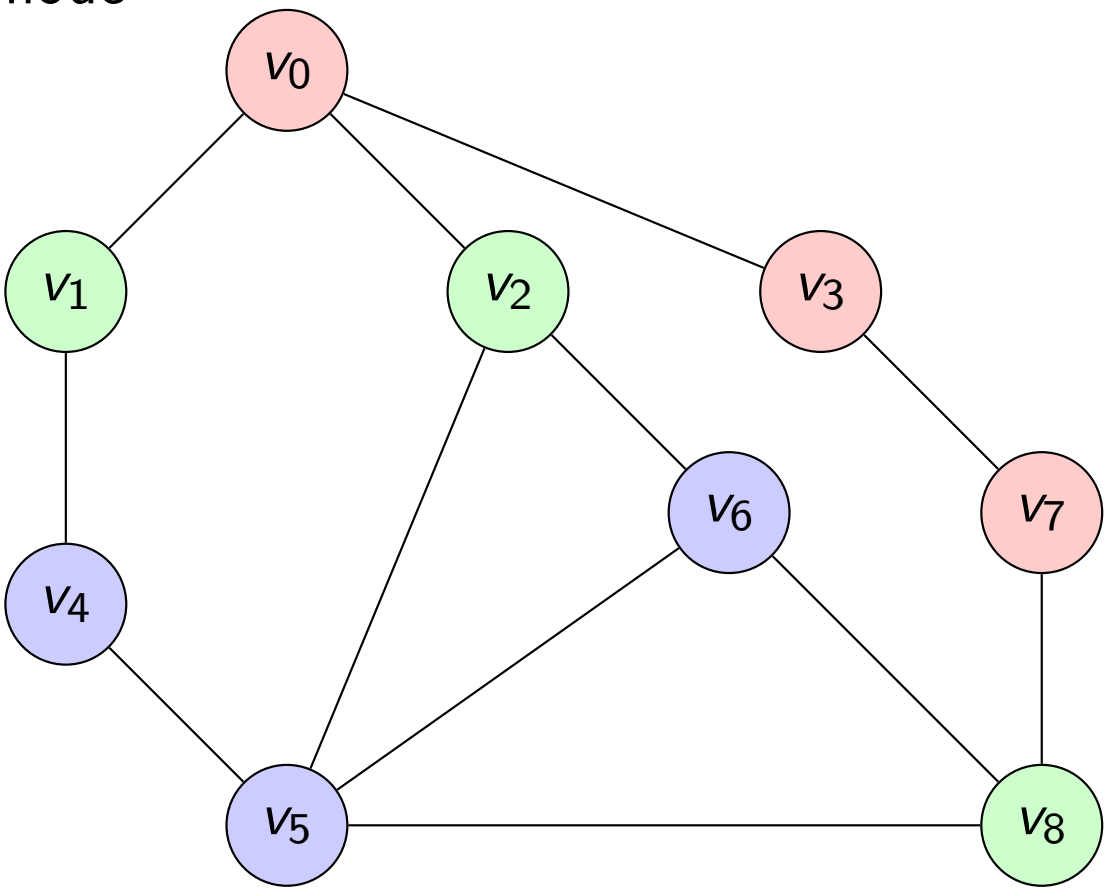

`Frontier` $= \{v_8, v_2, v_1\}$ and `explored` $= \{v_0, v_3, v_7\}$

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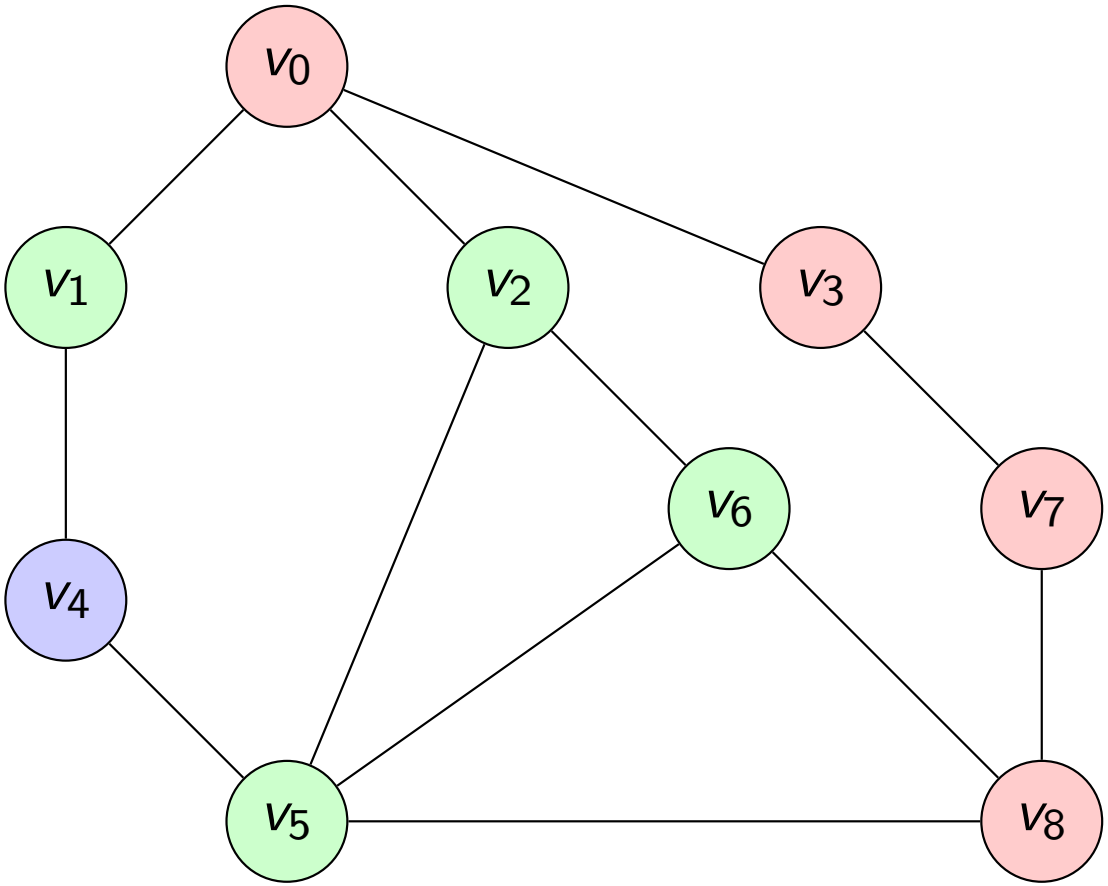

`Frontier` $= \{v_5, v_6, v_2, v_1\}$ and `explored` $= \{v_0, v_3, v_7, v_8\}$

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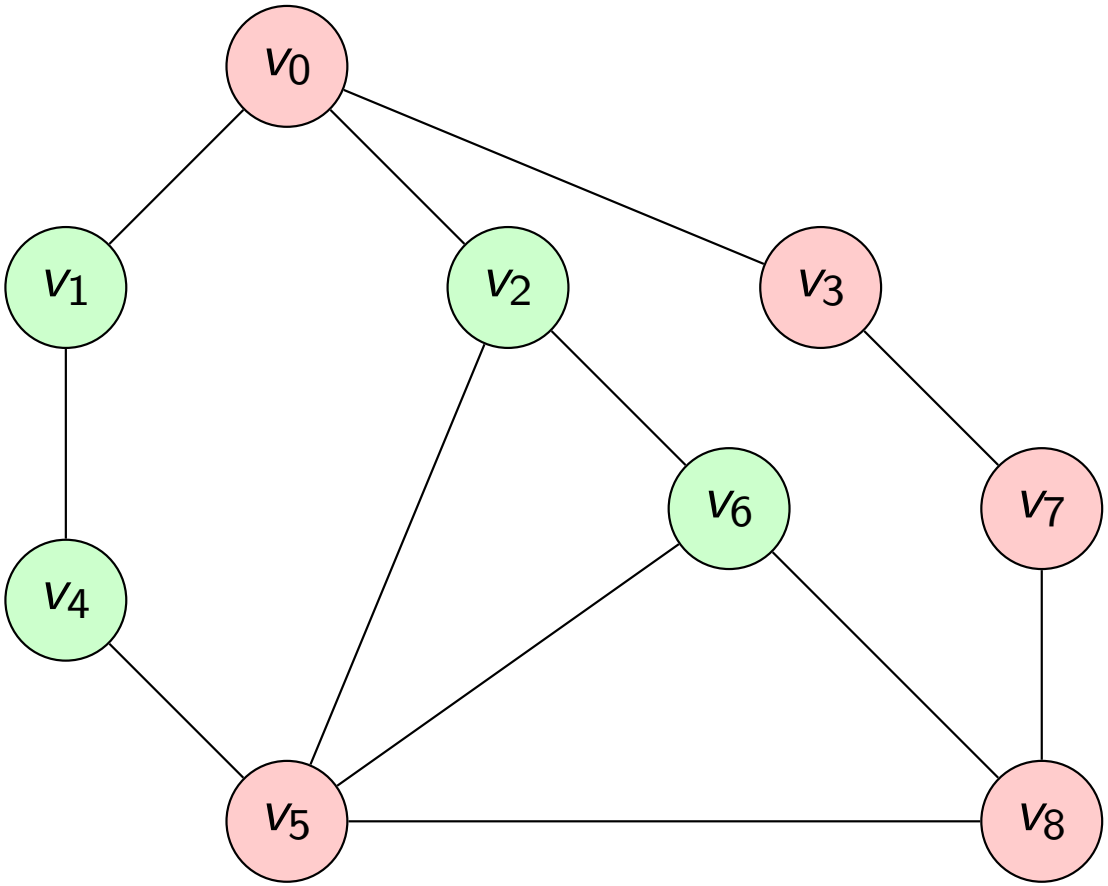

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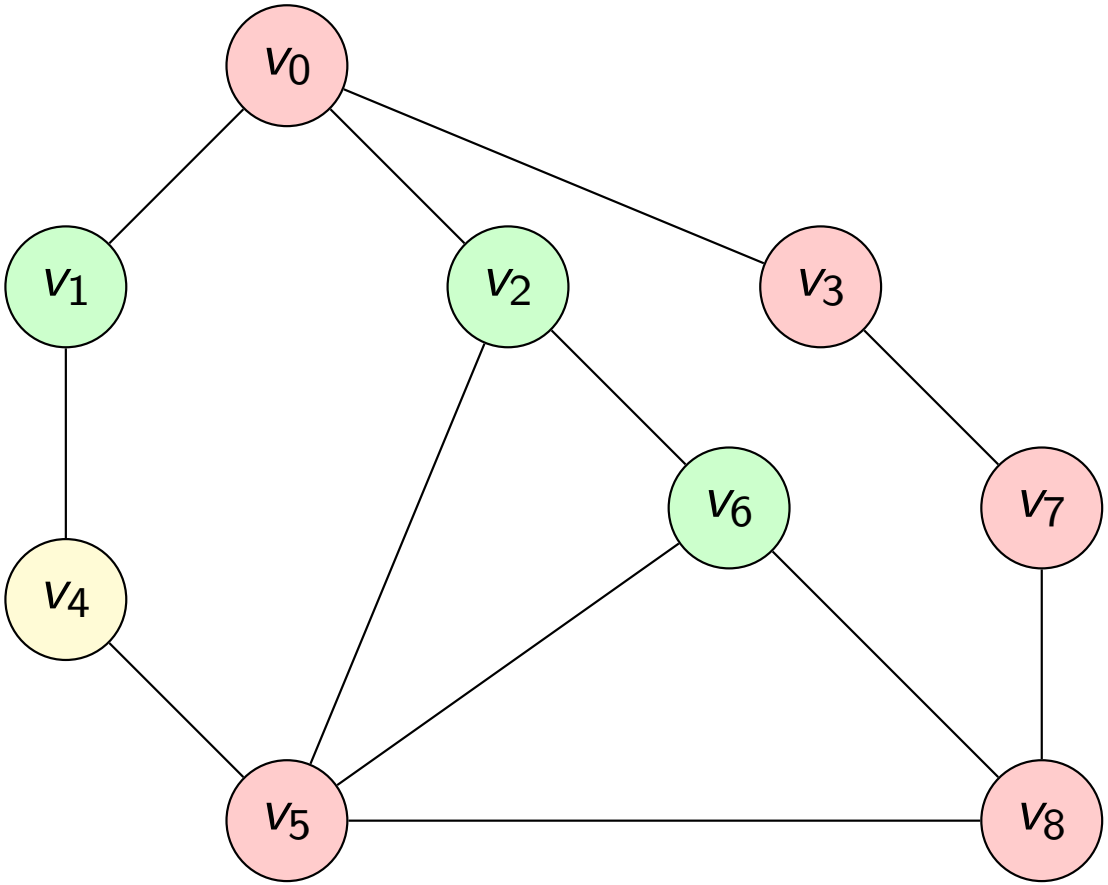

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- The order in which nodes are expanded is governed by the order they are pushed onto and popped from the stack. This depends on the order in which they are encountered in the "edge list".
- The search could be changed to be a *breadth first search* by using a *queue* instead of a stack to store the frontier nodes.

```
00  Algorithm pathbetween(G(V, E), v_i, v_j)
01      Let frontier be an empty stack
02      Let explored be an empty set
03      Push v_i onto frontier
04      Loop:
05          If frontier is empty
06              Return False
07          Else
08              current ← pop(frontier)
09              If current = v_j
10                  Return True
11              Else
12                  explored ← explored + current
13                  For all neighbours n of current
14                      If n ∉ frontier and n ∉ explored
15                          push n onto frontier
```

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- If an *adjacency list* representation is used to store the edges then the neighbours for each can be found directly and thus only $|E|$ edges in total need to be considered.
  So the algorithm would then be $\Theta(|V| + |E|)$ in the worst case

Good Luck with the exam

Post questions on the discussion forum if you have any.

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